24PE1003 STOCHASTIC MODELS (3-0-0)

Objective:

Course description Mathematical models based on probability theory prove to be extremely useful in describing and analyzing complex systems that exhibit random components. The goal of this course is to introduce several classes of stochastic processes, analyze their behavior over a finite or infinite time horizon, and help students enhance their problem solving skills.

The course combines classic topics such as martingales, Markov chains, renewal processes, and queuing systems with approaches based on Stein's method and on concentration inequalities.

The course focuses mostly on discrete-time models and explores a number of applications in operations research, finance, and engineering.

The course is directed towards graduate students who have a mathematically rigorous interest in stochastic processes and stochastic modelling.

MODULE – I

Preliminaries

- Expectation and integration
- Almost sure convergence and the dominated convergence theorem
- Convergence in probability and in distribution
- The law of large number and the ergodic theorem

Stein's method and central limit theorems

- Coupling
- Poisson approximation and Le Cam's theorem
- The Stein-Chen method
- Stein's method for the geometric and the normal distribution

MODULE – II

Conditional expectation and martingales

- Conditional expectation
- Martingales
- The martingale stopping theorem
- The Hoeffding-Azuma inequality
- The martingale convergence theorem
- Uniform integrability

Probability inequalities

- Jensen's inequality
- Probability bounds via the importance sampling identity
- Chernoff bounds
- Second moment and conditional expectation inequalities

MODULE – III

Discrete-time Markov chains

- Chapman-Kolmogorov equations and classification of states
- The strong Markov property
- Stationary and limiting distributions
- Transition among classes, the gambler's ruin problem, and mean times in transient states
- Branching processes
- Time reversibility

Renewal theory

- Limit theorems
- Renewal reward processes
- Blackwell's theorem

MODULE – IV

Queuing theory

- The Poisson process
- Queuing systems: M/M/1, M/G/1, G/M/1, and G/G/1

Outcome:

Students would acquire a rigorous understanding of basic concepts in probability theory. They would learn some important concepts concerning multiple random variables such as Bayes rule for random variables, conditional expectation and its uses etc. They would also learn stochastic processes, including Markov Chains and Poisson Processes. The course would provide the background needed to study topics such as Machine Learning, Adaptive Signal Processing, Estimation Theory etc.

Books Recommended:

1. Introduction to Algorithms, Thomas H.Corman, Charles E.Leiserson, Ronald L.Rivest, Second Edition, PHI 2003.
2. Data structures and Algorithm Analysis in C++, Mark Allen Weiss, PearsonEducation, 3rd Ed, 2007.
3. Online Computation and Competitive Analysis - A. Borodin and R. El-Yaniv,Cambridge Univ. Press, 1998.
4. Approximation Algorithms - Vijay V. Vazirani, Springer Verlag, 2003.
5. S.M. Ross and E.A. Peköz, A second course in probability, www.probabilitybookstore.com,Boston, MA, 2007.
6. S.M. Ross, Stochastic processes, John Wiley & Sons, Inc., New York, second edition, 1996
7. G.R. Grimmett and D.R. Stirzaker, Probability and random processes, Oxford University Press,New York, third edition, 2001, (same level).
8. G.F. Lawler, Introduction to stochastic processes, Chapman & Hall/CRC, Boca Raton, FL,second edition, 2006, (same level).
9. D. Williams, Probability with martingales, Cambridge University Press, Cambridge, 1991.